

# PUZZLING ASPECTS OF HOT QUANTUM FIELDS

T. GRANDOU

*Institut Non Linéaire de Nice, 1361 route des lucioles,  
06560 Valbonne, France*

*E-mail: grandou@inln.cnrs.fr*

Over the past decade, Finite Temperature Quantum Field Theories (FTQFT's) have benefitted from impressive developments, while an increasing number of intriguing points were made. Some of them are presented here, recent and older, in a non exhaustive list.

## 1 In the beginning..

Soon after the Asymptotic Freedom property of the strong running coupling constant of QCD was established, Collins and Perry<sup>1</sup> suggested that the same property holds true in ultra dense nuclear matter conditions as well. The idea was then quickly extended to conditions where the effects of not so high a matter density  $\mu$ , could be compensated for by a high enough temperature  $T$ , the whole picture boiling down to a diagram displaying the confined and deconfined phases of the QCD quanta. In the same line of reasoning, it is to be noted that, likewise, broken symmetries are expected to be restored in the high  $T$ -limit<sup>2</sup>. However, just the opposite scenario was also shown to be a possible outcome<sup>3</sup>!

## 2 An axiomatic point of view

One can show that FTQFT's can be renormalized *à la* BPHZ, and that, once equations of motion, standard axiomatic requirements (such as space time translation and rotational invariances, locality, cluster property, etc..), and the KMS conditions specific to thermal equilibrium are imposed, then, the perturbative expansion of the (Grand Canonical) thermal correlation functions

$$W(x_1, x_2, \dots, x_n) = \langle \varphi(x_1) \varphi(x_2) \dots \varphi(x_n) \rangle_\beta$$

enjoy a unique determination, and allow a reconstruction of the full representation space of the field algebra by means of the GNS construction<sup>4</sup>. On the other hand, these thermal correlation functions may as well not exist at all in view of Infra-Red (IR) non (Lebesgue)-integrable singularities related to vanishing external momenta, and/or exceptional combinations of them! Such IR non integrable singularities have effectively been met in the course of actual perturbative calculations<sup>5</sup>. A natural question is thus: could these IR non

integrable singularities be compensated by other diagrams, as thermal Gauge Field Theories have proven to be pretty rife with cancellations? In the case of two point functions, for example, it has been shown recently that for a large enough variety of self energies and vertices, self energy insertions along internal lines cancel against insertions of *rungs* in ladder type diagrams<sup>6</sup>. However this nice and simplifying mechanism is effective in some kinematical regimes only where a particular resolution of a Schwinger-Dyson equation for the vertex, in terms of the self energy function, is available. What about other phase space regimes? And what about the calculation of processes related to thermal correlation functions with a higher number of external momenta, some of them exceptional?

### 3 Perturbative Regimes

Analyzing the representation space of the GNS construction, it has been shown that (due to Lorentz invariance explicit thermal breaking) FTQFT's are inherently non perturbative in the following sense: there are no *LSZ*-asymptotic states in term of which to devise any kind of *S*-matrix approach<sup>7</sup>. Only quasiparticle states could be used for that purpose, but unfortunately, such attempts quickly become practically untractable. This, however, is just one particular realization of "*Non-Perturbativeness*" a notion which, being negative, opens up over other aspects and realizations, whose inter-relations are not necessarily well understood or even investigated. A more immediate aspect is of course related to the smallness of the effective running coupling constant. In this respect, a large number of calculations of  $\alpha_s(\mu, T)$ , the dependance on the temperature  $T$  of the effective running strong coupling of QCD, have been performed, with rather odd results. Different from each others, the results concerning the GellMann-Low  $\beta_T$ -function, can all be written as

$$\beta_T(\alpha_s(\mu, T)) = T \frac{\partial}{\partial T} \left( \frac{g_R^2}{4\pi} = \alpha_s(\mu, T) \right) = -C(T) \alpha_s^2(\mu, T) \quad (1)$$

where  $\mu$  stands for some  $T = 0$  renormalization scale. The prefactors  $C(T)$  came out dependent, (i): on the vertex choosen to define the renormalized coupling (tri-or quadri-linear in the gauge fields, gauge-quarks, ghost-gauge vertex), (ii): on the external momenta vertex configurations, (iii): .. and on the gauge choice itself! No meaning could accordingly be attached to these results. The most satisfying effort produced in order to get rid of the above drawbacks is certainly the one having made use of the so called Vilkovisky-DeWitt effective action,  $\Gamma_{VD}$ , which, contrarily to the conventional effective action of QCD<sup>8</sup>, is explicitly gauge invariant and independent of the condition

chosen to quantize the theory. An unambiguous procedure results where the renormalized coupling normalization conditions are entirely fixed by gauge invariance itself, and any drawback (i) to (iii) is consistently circumvented. Now, the result can essentially be written as

$$\beta_T(\alpha_s) = - \left( b - \frac{21}{6} \pi^2 N_C \left( \frac{T}{\mu} \right) + \mathcal{O} \left( \left( \frac{T}{\mu} \right)^{-2} \right) \right) \alpha_s^2 \quad (2)$$

where  $b = (11N_C - 2N_f)/6$ . For large enough  $T$ 's, the  $\beta_T$ -function is positive, and, in contrast with Lattice data as well as physical intuition, Asymptotic Freedom at high  $T$  (and/or density) cannot be deduced from one loop Renormalization Group arguments! It seems hard to believe that the one loop specificity could be at fault in this somewhat disappointing result, and more satisfying to keep in mind that  $\Gamma_{VD}$  is not, itself, free of any drawback. For example, its expansion in terms of mean fields  $\overline{A}$  (and  $\overline{A}_0$ , in the absence of sources) is

$$\Gamma_{VD}(\overline{A}) = \Sigma \frac{1}{n!} \Gamma_{VD}^{(n)}{}^{i_1 \dots i_n} (\overline{A}_{i_1} - \overline{A}_{0i_1}) \dots (\overline{A}_{i_n} - \overline{A}_{0i_n}) \quad (3)$$

and the coefficients,  $\Gamma_{VD}^{(n)}$  cannot be interpreted in terms of 1PI-correlation function of  $A$  or of any other operator. This is not the case in a Wilsonian flow approach of hot QCD, where gauge invariance can also be maintained along the flow

$$\delta_\alpha \{ \Gamma_{k,T}(\overline{A}) - \Gamma_{k,T=0}(\overline{A}) \} = 0 \quad (4)$$

where the cutoff  $k$  is the flow parameter<sup>9</sup>. Certainly, it would be interesting to see the results one would get for  $\beta_T$ .

#### 4 The Resummation Program

From now on, we assume a small enough  $\alpha_s$ . For Green's functions with soft, order  $gT$ -external momenta, a re-organization of bare Perturbation Theory (PT) is mandatory to get the completeness of leading thermal corrections. This is achieved through a Resummation Program (RP)<sup>10</sup>, a beautiful effective Perturbation Theory, ruling soft scale fluctuations, and a leading order approximation scheme, fully consistent with gauge invariance. In particular, the RP has solved the static gluon damping rate problem, and thanks to the *Dynamical Screening Mechanism*<sup>11</sup>, seems to have improved the IR sector of bare PT. It is however from damping rate's calculations that two major obstructions to the RP came about, with first<sup>12</sup>, the moving fermion damping

rate of QED and QCD. In effect, when the latter is evaluated by the fermion mass shell, a logarithmic IR singularity is found in both QED and QCD cases

$$\gamma(E, p)|_{E=p} = \frac{e^2 T}{2\pi} \int_{\frac{|E-p|}{2}}^{k^*} \frac{k dk}{k^2} + \text{regular} \quad (5)$$

where the fermion has been taken massless for the sake of simplicity, while the same holds true for massive fermions too. This IR singularity is due to unscreened transverse modes, as can be read off the intermediate energy sum rule leading to (5)

$$\int_{-k}^{+k} \frac{dk_0}{2\pi k_0} {}^* \rho_T(k_0, k) \simeq \frac{1}{k^2} + \mathcal{O}\left(\frac{1}{m_D^2}\right), \quad k/m_D \ll 1, \quad m_D^2 = \mathcal{O}(g^2 T^2) \quad (6)$$

where  ${}^* \rho_T$  is the transverse spectral density associated to the RP's effective propagator (8). A further (Bloch-Nordsieck type) resummation has been proposed in order to screen this IR singular behaviour<sup>13</sup>. A possible transverse screening mass has also been looked for in QCD, with some encouraging result of order  $g^2 T$ , in the case of a three dimensional  $SU(N_C)$  theory<sup>14</sup>. However it has been recently argued that such a mass could be too small to act as an efficient IR cutoff in the case of QCD<sup>15</sup>, and eventually, no such mass can be invoked for QED. One may therefore wonder how the problem could get translated in another resummation scheme of the thermal leading effects, as we will comment shortly. The emission rate of a soft real photon from a quark-gluon plasma has also been found troublesome<sup>16</sup>, since, up to regular terms, one gets a result affected with a collinear singularity (the dimensional  $\varepsilon$  parameter regularizes the divergence)

$$\frac{C^{st}}{\varepsilon} \int \frac{d^D P}{(2\pi)^D} \delta(\hat{Q} \cdot P) (1 - 2n_F(p_0)) \sum_{s=\pm 1, R=P, P+Q} \pi(1 - s \frac{r_0}{r}) \beta_s(R) \quad (7)$$

where  $\beta_s(R)$  is related to the fermionic effective propagator. A so-called *Thermal Asymptotic Mass*,  $m_\infty$ , has been proposed a gauge invariant way<sup>17</sup>, of order  $gT$ , which takes the original  $1/\varepsilon$  of (7) to a large logarithm  $\ln(1/g)$ . However the method, if consistent, does not really save the general situation as we will see shortly, so that again, one may wonder about what could come out of another resummation scheme of the thermal leading effects. We come to this point now.

## 5 Perturbative Resummation

A Perturbative Resummation Scheme of the thermal leading effects can be introduced<sup>18</sup>, differing the usual RP, only the effective propagators. Instead

of effective functions like

$${}^*\Delta_{\alpha\alpha}(K) = \frac{i}{K^2 - \Pi_{\alpha\alpha}^{HTL}(k_0, k) + i\varepsilon_\alpha} \quad (8)$$

with  $\alpha$  labelling the two Retarded/Advanced field types of a (R/A) real time formalism for example, and with the superscript *HTL* to mean the leading part of  $\Pi$ , order  $g^2T^2$ , we use the geometrical series representations

$$\sum_{N=0}^{\infty} \Delta_{\alpha\alpha}^{(N)}(k_0, k) = i \sum_{N=0}^{\infty} (\Delta_{\alpha\alpha}^{(0)}(k_0, k))^{N+1} (\Pi_{\alpha\alpha}^{HTL}(k_0, k))^N \quad (9)$$

effective vertices remaining the same in both RP and Perturbative Resummation scheme. The effective functions (8) and (9) satisfy the same Dyson equation,

$${}^*\Delta_{\alpha\alpha}(k_0, k) = \Delta_{\alpha\alpha}^{(0)}(k_0, k) + \Delta_{\alpha\alpha}^{(0)}(k_0, k) \Pi_{\alpha\alpha}^{HTL}(k_0, k) {}^*\Delta_{\alpha\alpha}(k_0, k) \quad (10)$$

but are definitely different one dimensional  $k_0$ -distributions. When used in the course of practical calculations, they lead to irreducibly different IR behaviours, and this feature appears specific to the thermal context<sup>18</sup>. While (9) is by construction a Taylor series in  $g^2$ , (8) is not; this is why the sum rule (6) for example, can be seen to display a Laurent series behaviour in  $g^2$ . In the moving fermion damping rate calculation, the outcome is that the series associated to longitudinal degrees is analytic in a domain whose real restriction is given by  $|\vec{k}| \geq m_D$ , whereas transverse degrees of freedom rigorously do not contribute

$$\sum_{N=0}^{\infty} \int_{-k}^{+k} \frac{dk_0}{\pi k_0} \left(1 - \frac{k_0^2}{k^2}\right) \text{disc}_{p_0} \Delta_{\alpha\alpha}^{(N)}(k_0, k) = 0 \quad (11)$$

There are no unscreened magnetostatic modes problem in this resummation scheme, whereas the same IR singularity as displayed in (5) results of a continuation to  $0 \leq k \leq m_D$  of the longitudinal series; and the same features here equally apply to both QED and QCD. For the soft real photon emission rate, or second obstruction (7), a very different setting of the problem is obtained also within the Perturbative Resummation scheme<sup>19</sup>.

## 6 Completeness

Though collinear divergences are effectively regularized by the introduction of a *Thermal Asymptotic Mass*, the latter, we wrote, "does not save". This is because of the *Collinear Enhancement Mechanism* discovered a few years

ago<sup>20</sup>. This mechanism shows up in the calculation of processes related to Green's functions with external momenta on the light cone, and makes higher number of loops contributions as large, or even larger than lower ones, thus compromising the RP's completeness. One has symbolically, with  $\hat{K}$  denoting the light-like vector  $(1, \hat{k})$

$$\int \frac{d\hat{K}}{4\pi} \frac{\mathcal{F}(\hat{K}, P, R, ..)}{(\hat{K} \cdot P - \frac{m_\infty^2}{2k})(\hat{K} \cdot R - \frac{m_\infty^2}{2k}) .. ( .. )} \longrightarrow \left(\frac{1}{g}\right)^n F(P, R, ..) \quad (12)$$

This in the end, should not come out as a too big surprise if we keep in mind that, first devised in the imaginary time formalism, the RP's power counting analysis implicitly assumed angular integrals on the order of unity! Now, if we are more or less used to the idea that the RP breaks down by the light cone, we may be not so familiar with the idea that it could be so elsewhere too. However, if we take the static photon production rate out of a Quark-Gluon Plasma, we find the one loop result<sup>21</sup>

$$\text{Im}\Pi_R(q_0, \vec{0}) = -\frac{e^2 g^2 N_c C_F}{32\pi} q_0 T \ln \left( \frac{2q_0 T}{\text{Max}\{q_0^2, m_F^2\}} \right) \quad (13)$$

where  $m_F^2 = g^2 T^2 C_F / 8$  is the fermionic thermal mass. Now, as shown more recently<sup>22</sup>, a two loop contribution is

$$\text{Im}\Pi_R(q_0, \vec{0}) = -\frac{e^2 g^2 N_c C_F}{32\pi} q_0 T \ln \left( \frac{2q_0 T}{q_0^2 + m_g^2} \right) \quad (14)$$

where  $m_g$  is the gluonic Debye mass of order  $gT$ . The striking aspect of (13) and (14) is of course that the same orders of magnitude and dependences on  $q_0$  and  $T$  are obtained at one and two loops, and similar contributions will also have to be looked for at any arbitrary higher number of loops. This new RP's breakdown takes place out of the light cone region.

## 7 Low energy limit

To end up with these Hot Quantum Fields somewhat puzzling aspects, let us mention that the low energy limit of Hot Gauge Theories (that is, at an energy scale on the order of  $g^2 T$  at most), expected to be both local and stochastic, has effectively been found so in a series of recent papers<sup>23</sup>. This however holds true at a leading-logarithm level of approximation, while lattice calculations would rather indicate very large subleading-logarithm corrections<sup>24</sup>!

## 8 Conclusion

Compared to their ordinary vacuum representations, the quantized field algebras-*KMS* (thermal) representations display an unexpected richness of new structures and results, all of them being eventually generated by the interplay of only two (non decorrelated) major features: (i) The Lorentz invariance thermal breaking, and, (ii) The appearance of a dimensionful parameter in the formal perturbative series, namely the temperature. The few puzzling points which we have listed here, display the surprising richness of the subject as well as its rather unexpected difficulties. It may be, as some authors are inclined to think, that we are running short of some deeper conceptual understanding of FTQFT's. In any case, we are certainly lacking clear cut experimental results that would help us knowing whether we are going the right way or not .. including the very first of all steps, implementing the thermodynamical temperature on the basis of a formal analogy with an imaginary time<sup>25</sup>

$$it/\hbar \longleftrightarrow 1/k_B T \quad (15)$$

though it is not impossible either, that this formal analogy itself relies on a much deeper and fundamental duality relation between time and absolute temperature<sup>26</sup>, .. a possibility which would certainly stand for one more intriguing aspect of FTQFT's!

## Acknowledgments

I wish to express my gratitude to H.M. Fried and B. Müller for having organized this fifth QCD workshop.

## References

1. J.C. Collins and M.J. Perry, *Phys. Rev. Lett.* **34** (1975) 1353.
2. A.D.Linde, *Rep. Prog. Phys.* **42** (1979) 389; L. Dolan and R. Jackiw, *Phys. Rev.* **D9** (1974) 3320.
3. S. Weinberg, *Phys. Rev.* **D9** (1974) 3357; R.D. Pisarski, in Workshop on Hot QCD, Winnipeg, July 1992.
4. O. Steinmann, *Commun. Math. Phys.* **170** (1995) 405.
5. T. Grandou, M. Le Bellac and D. Poizat, *Nucl. Phys.* **B358** (1991) 408;
6. V.V. Lebedev and A.V. Smilga, *Ann. Phys.(N.Y.)* **202** (1990) 229; *Physica A* **181** (1992) 187; M.E. Carrington, R. Kobes and E. Petitgirard, *Phys. Rev.* **D57** (1998) 2631; M.E. Carrington, R. Kobes, *Phys. Rev.* **D57** (1998) 6372.
7. N.P. Landsman, *Ann. Phys.(N.Y.)* **186** (1988) 141.

8. A.K. Rebhan, *Z. Phys. C* **B30** (1986) 309.
9. D.F. Litim and J.M. Pawłowski, preprint DIAS-STP-99-01, ECM-UB-PF-99-02.
10. E. Braaten and R. Pisarski, *Phys. Rev. Lett.* **64** (1990) 1338; *Nucl. Phys. B* **337** (1990) 569; J. Frenkel and J.C Taylor, *Nucl. Phys. B* **334** (1990) 199.
11. M. Le Bellac, “Thermal Field Theory”, Cambridge University Press, 1996.
12. R.D. Pisarski, *Phys. Rev. Lett.* **63** (1989) 1129.
13. J.P. Blaizot and E. Iancu, *Phys. Rev. D* **55** (1997) 973; *Phys. Rev. D* **56** (1997), 7877.
14. D. Karabali, C. Kim and V.P. Nair, *Phys. Lett. B* **434** (1998) 103; *Nucl. Phys. B* **524** (1998) 661; J. Reinbach and H. Schulz, *Phys. Lett. B* **467** (1999) 247.
15. P. Aurenche, F. Gelis and H. Zaraket, preprint LAPTH-756/99, BNL-NT-99/5.
16. R. Baier, S. Peigné and D. Schiff, *Z. Phys. C* **62** (1994) 337.
17. F. Flechsigs and A.K. Rebhan, *Nucl. Phys. B* **464** (1996), 279.
18. B. Candelpergher and T. Grandou, preprint INLN 99/22; Proceedings of the IV Workshop on Quantum Chromodynamics, World Scientific, 1999.
19. T. Grandou, in preparation.
20. P. Aurenche, F. Gelis, R. Kobes and E. Petitgirard, *Phys. Rev. D* **54** (1996) 5274.
21. E. Braaten, R.D. Pisarski and T.C. Yuan, *Phys. Rev. Lett.* **64** (1990) 2242.
22. P. Aurenche, F. Gelis, R. Kobes and H. Zaraket, *Phys. Rev. D* **58** (1998) 085003.
23. E. Iancu, *Phys. Lett. B* **435** (1998) 152; D. Bödeker, *Phys. Lett. B* **426** (1998) 351; P. Arnold, D. Son and L.G. Yaffe, *Phys. Rev. D* **55** (1997) 6264.
24. G.D. Moore, C.R. Hu and B. Müller, *Phys. Rev. D* **58** (1998) 045001.
25. F. Bloch, *Z. Phys.* **74** (1932) 295.
26. W. Królikowski, *Acta. Physica. Polonica. B* **29** (1998) 2081.